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TERM 3 MATHS

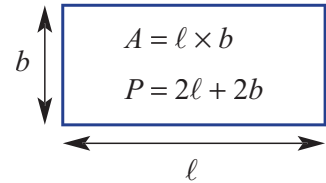
Week	9	Year	8
Student's Name			

2G Formulas and relationships

EXTENSION



Some equations are called formulas. A formula shows the relationship between two or more variables. For example, you know from measurement that the area of a rectangle is related to its length and breadth, given by the formula $A = \ell \times b$ and its perimeter is given by $P = 2\ell + 2b$.



Although these are often used as a definition for the area and a definition for the perimeter, they are also just equations – two expressions written on either side of an equals sign.

Let's start: Rectangular dimensions

You know that the area and perimeter of a rectangle are given by $A = \ell \times b$ and $P = 2\ell + 2b$.

- If $\ell = 10$ and $b = 7$, find the perimeter and the area.
- If $\ell = 2$ and $b = 8$, find the perimeter and the area.
- Notice that sometimes the area value is bigger than the perimeter value and sometimes the area value is less than the perimeter value. If $\ell = 10$, is it possible to make the area and the perimeter values equal?
- If $\ell = 2$, can you make the area and the perimeter equal? Discuss.

Key ideas

- A **formula** or **rule** is an equation containing two or more pronumerals, one of which is the subject of the equation.
- The **subject** of a formula is a pronumeral that occurs by itself on the left-hand side, e.g. V is the subject of $V = 3x + 2y$.
- To use a formula substitute all the known values and then solve the equation to find the unknown value.

Example 11 Applying a formula

Apply the formula for a rectangle's perimeter $P = 2\ell + 2b$ to find:

a P when $\ell = 4$ and $b = 7$

b ℓ when $P = 40$ and $b = 3$

SOLUTION

a $P = 2\ell + 2b$
 $P = 2 \times 4 + 2 \times 7$
 $P = 22$

b $P = 2\ell + 2b$
 $40 = 2\ell + 2 \times 3$

$$\begin{array}{r} 40 = 2\ell + 6 \\ -6 \qquad \qquad -6 \\ \hline 34 = 2\ell \\ \div 2 \qquad \qquad \div 2 \\ \hline 17 = \ell \end{array}$$

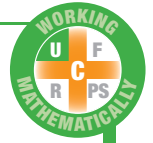
EXPLANATION

Write the formula.
 Substitute in the values for ℓ and b .
 Simplify the result.

Write the formula.
 Substitute in the values for P and b to obtain an equation.

Solve the equation to obtain the value of ℓ .

Exercise 2G EXTENSION



- 1
- Substitute $x = 4$ into the expression $x + 7$.
 - Substitute $a = 2$ into the expression $3a$.
 - Substitute $p = 5$ into the expression $2p - 3$.
 - Substitute $r = -4$ into the expression $7r$.
- 2 If you substitute $P = 10$ and $x = 2$ into the formula $P = 3m + x$, which of the following equations would you get?
- | | |
|------------------------|------------------------|
| A $10 = 6 + x$ | B $10 = 3m + 2$ |
| C $2 = 3m + 10$ | D $P = 30 + 2$ |
- 3 If you substitute $k = 10$ and $L = 12$ into the formula $L = 4k + Q$, which of the following equations would you get?
- | | |
|-------------------------|------------------------|
| A $12 = 40 + Q$ | B $L = 40 + 12$ |
| C $12 = 410 + Q$ | D $10 = 48 + Q$ |

Example 11a

- 4 Consider the rule $A = 4p + 7$.

- | | |
|---------------------------------|---|
| a Find A if $p = 3$. | b Find A if $p = 11$. |
| c Find A if $p = -2$. | d Find A if $p = \frac{13}{2}$. |

Example 11b

- 5 Consider the rule $U = 8a + 4$.

- Find a if $U = 44$. Set up and solve an equation.
- Find a if $U = 92$. Set up and solve an equation.
- If $U = -12$, find the value of a .

- 6 Consider the relationship $y = 2x + 4$.

- Find y if $x = 3$.
- By solving an appropriate equation, find the value of x that makes $y = 16$.
- Find the value of x if $y = 0$.

- 7 Use the formula $P = mv$ to find the value of m when $P = 22$ and $v = 4$.

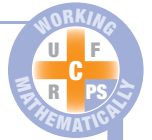
- 8 Assume that x and y are related by the equation $4x + 3y = 24$.

- If $x = 3$, find y by solving an equation.
- If $x = 0$, find the value of y .
- If $y = 2$, find x by solving an equation.
- If $y = 0$, find the value of x .

- 9 Consider the formula $G = k(2a + p) + a$.

- If $k = 3$, $a = 7$ and $p = -2$, find the value of G .
- If $G = 78$, $k = 3$ and $p = 5$, find the value of a .





10 The cost \$ C to hire a taxi for a trip of length d km is $C = 3 + 2d$.

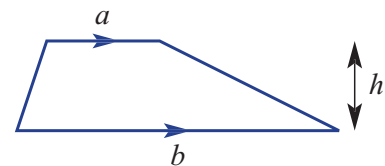
- Find the cost of a 10 km trip (i.e. for $d = 10$).
- A trip has a total cost of \$161.
 - Set up an equation by substituting $C = 161$.
 - Solve the equation systematically.
 - How far did the taxi travel? (Give your answer in km.)

11 In Rugby Union, 5 points are awarded for a try, 2 points for a conversion and 3 points for a penalty. A team's score is therefore $S = 5t + 2c + 3p$, where t is the number of tries, c is the number of conversions and p is the number of penalties.

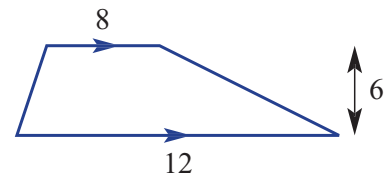
- Find the score if $t = 2$, $c = 1$ and $p = 2$.
- If a team's score is 20 and they scored 3 tries and 1 penalty, how many conversions did they score?
- In Rugby League, 4 points are awarded per try (t), 2 for a conversion (c) and 1 for a drop goal (d). Write a formula for the total score (S) of a Rugby League team.



12 The formula for the area of a trapezium is $A = \frac{1}{2}h(a + b)$.

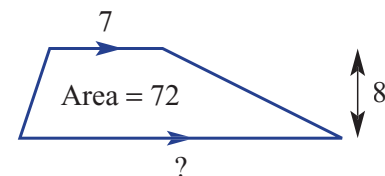


a Find the area of the trapezium shown to the right.



b Find the value of h if $A = 20$, $a = 3$ and $b = 7$.

c Find the missing value in the trapezium shown to the right.



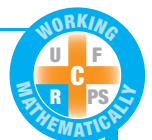
13 Katy is a scientist who tries to work out the relationship between the volume of a gas, V mL, and its temperature $T^\circ\text{C}$. She makes a few measurements.

V	10	20
T	10	15

- What is a possible rule between V and T ?
- Use your rule to find the volume at a temperature of 27°C .
- Prove that the rule $T = \frac{(V - 10)^2}{20} + 10$ would also work for Katy's results.

14 Consider the rule $G = 120 - 4p$.

- If p is between 7 and 11, what is the largest value of G ?
- If p and G are equal, what value do they have?





- 15 Marie is a scientist who is trying to discover the relationship between the volume of a gas V , its temperature T and its transparency A . She makes a few measurements.

	Test 1	Test 2
V	10	20
A	2	5
T	15	12



Which one or more of the following rules are consistent with the experiment's results?

- A** $T = \frac{3V}{A}$ **B** $T = V + 2A$
C $T = 17 - A$
- 16 Temperatures in degrees Fahrenheit and Celsius are related by the rule $F = 1.8C + 32$.
- a** By substituting $F = x$ and $C = x$, find a value such that the temperature in Fahrenheit and the temperature in Celsius are equal.
- b** By substituting $F = 2x$ and $C = x$, find a temperature in Celsius that doubles to give the temperature in Fahrenheit.
- c** Prove that there are no Celsius temperatures that can be multiplied by 1.8 to give the temperature in Fahrenheit.

Enrichment: Mobile phone plans

- 17 Two companies have mobile phone plans that factor in the number of minutes spent talking each month (t) and the total number of calls made (c).

Company A's cost in cents: $A = 20t + 15c + 300$

Company B's cost in cents: $B = 30t + 10c$

- a** In one month 12 calls were made, totalling 50 minutes on the phone. Find the cost in dollars that company A and company B would have charged.
- b** In another month, a company A user was charged \$15 (1500 cents) for making 20 calls. How long were these calls in total?
- c** In another month, a company B user talked for 60 minutes in total and was charged \$21. What was the average length of these calls?
- d** Briony notices one month that for her values of t and c , the two companies cost the same. Find a possible value of t and c that would make this happen.
- e** Briony reveals that she made exactly 20 calls for the month in which the two companies' charges would be the same. How much time did she spend talking?



2H Applications

EXTENSION



Although knowing how to solve equations is useful, it is important to be able to recognise when real-world situations can be thought of as equations. This is the case whenever it is known that two values are equal.

**Let's start: Sibling sum**

John and his elder sister are 4 years apart in their ages.

- If the sum of their ages is 26, describe how you could work out how old they are.
- Could you write an equation to describe the situation above, if x is used for John's age?
- How would the equation change if x is used for John's sister's age instead?



The difference in two people's ages can be expressed as an equation.

Key ideas

- An equation can be used to describe any situation in which two values are equal.
- To solve a problem follow these steps.
 - 1 Define pronumerals to stand for unknown numbers (e.g. let j = John's current age).
 - 2 Write an equation to describe the situation.
 - 3 Solve the equation systematically, if possible, or by inspection.
 - 4 Ensure you answer the original question, including the correct units (e.g. dollars, years, cm).
 - 5 Check that your solution is reasonable and makes sense.

Example 12 Solving a problem using equations

The weight of 6 identical books is 1.2 kg. What is the weight of one book?

SOLUTION

Let b = weight of one book.

$$6b = 1.2$$

$$\begin{array}{ccc} & 6b = 1.2 & \\ \div 6 & \curvearrowright & \div 6 \\ & b = 0.2 & \end{array}$$

The books weigh 0.2 kg each, or 200 g each.

EXPLANATION

Step 1: Define a pronumeral to stand for the unknown number.

Step 2: Write an equation to describe the situation.

Step 3: Solve the equation.

Step 4: Answer the original question. It is not enough to give a final answer as 0.2; this is not the weight of a book, it is just a number.

Example 13 Solving a harder problem using equations

Purchasing 5 apples and a \$2.40 mango costs the same as purchasing 7 apples and a mandarin that costs 60 cents. What is the cost of each apple?

SOLUTION

Let c = cost of one apple in dollars.

$$5c + 2.4 = 7c + 0.6$$

$$\begin{array}{ccc} & 5c + 2.4 = 7c + 0.6 & \\ -5c & \swarrow \quad \searrow & -5c \\ & 2.4 = 2c + 0.6 & \\ -0.6 & \swarrow \quad \searrow & -0.6 \\ & 1.8 = 2c & \\ \div 2 & \swarrow \quad \searrow & \div 2 \\ & 0.9 = c & \end{array}$$

Apples cost 90 cents each.

EXPLANATION

Step 1: Define a pronumeral to stand for the unknown number.

Step 2: Write an equation to describe the situation. Note that 60 cents must be converted to \$0.6 to keep the units the same throughout the equation.

Step 3: Solve the equation.

Step 4: Answer the original question. It is not enough to give a final answer as 0.9; this is not the cost of an apple, it is just a number.

Example 14 Solving problems with two related unknowns

Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.

SOLUTION

Let l = Luke's age.

$$l + 2l = 60$$

$$\begin{array}{ccc} & 3l = 60 & \\ \div 3 & \swarrow \quad \searrow & \div 3 \\ & l = 20 & \end{array}$$

Luke is 20 years old and Jane is 40 years old.

EXPLANATION

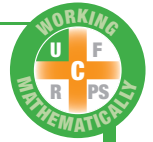
Step 1: Define a pronumeral for the unknown value. Once Luke's age is found, we can double it to find Jane's age.

Step 2: Write an equation to describe the situation. Note that Jane's age is $2l$ because she is twice as old as Luke.

Step 3: Solve the equation by first combining like terms.

Step 4: Answer the original question.

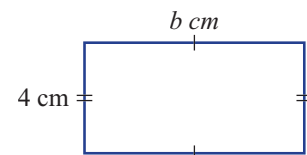
Exercise 2H EXTENSION



- 1 Match each of these worded descriptions with an appropriate expression.
- | | |
|---|------------------|
| a The sum of x and 3 | A $2x$ |
| b The cost of 2 apples if they cost $\$x$ each | B $x + 1$ |
| c The cost of x oranges if they cost $\$1.50$ each | C $3x$ |
| d Triple the value of x | D $x + 3$ |
| e One more than x | E $1.5x$ |
- 2 For the following problems choose the equation to describe them.
- a** The sum of x and 5 is 11.
- | | |
|-----------------------|-----------------------|
| A $5x = 11$ | B $x + 5 = 11$ |
| C $x - 5 = 11$ | D $11 - 5$ |
- b** The cost of 4 pens is $\$12$. Each pen costs $\$p$.
- | | |
|--------------------|--------------------|
| A $4 = p$ | B $12p$ |
| C $4p = 12$ | D $12p = 4$ |
- c** Josh's age next year is 10. His current age is j .
- | | |
|-----------------------|-----------------------|
| A $j + 1 = 10$ | B $j = 10$ |
| C 9 | D $j - 1 = 10$ |
- d** The cost of n pencils is $\$10$. Each pencil costs $\$2$.
- | | |
|--------------------------|----------------------|
| A $n \div 10 = 2$ | B 5 |
| C $10n = 2$ | D $2n = 10$. |
- 3 Solve the following equations.
- | | | | |
|--------------------|------------------------|-------------------------|------------------------|
| a $5p = 30$ | b $5 + 2x = 23$ | c $12k - 7 = 41$ | d $10 = 3a + 1$ |
|--------------------|------------------------|-------------------------|------------------------|

Example 12

- 4 Jerry buys 4 cups of coffee for $\$13.20$.
- Choose a pronumeral to stand for the cost of one cup of coffee.
 - Write an equation to describe the problem.
 - Solve the equation systematically.
 - Hence state the cost of one cup of coffee.
- 5 A combination of 6 chairs and a table costs $\$3000$. The table alone costs $\$1740$.
- Define a pronumeral for the cost of one chair.
 - Write an equation to describe the problem.
 - Solve the equation systematically.
 - Hence state the cost of one chair.
- 6 The perimeter of this rectangle is 72 cm.
- Write an equation to describe the problem, using b for the breadth.
 - Solve the equation systematically.
 - Hence state the breadth of the rectangle.

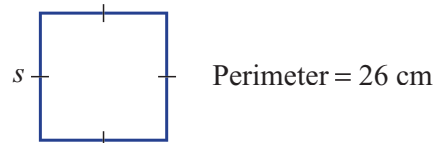


- 7** A plumber charges a \$70 call-out fee and \$52 per hour. The total cost of a particular visit was \$252.
- Define a pronumeral to stand for the length of the visit in hours.
 - Write an equation to describe the problem.
 - Solve the equation systematically.
 - State the length of the plumber's visit, giving your answer in minutes.



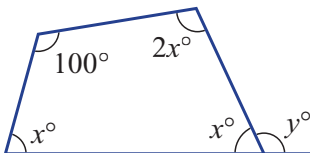
Example 13 **8** A number is tripled, then 2 is added. This gives the same result as if the number were quadrupled. Set up and solve an equation to find the original number.

- 9** A square has a perimeter of 26 cm.
- Solve an equation to find its side length.
 - Hence state the area of the square.



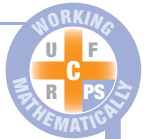
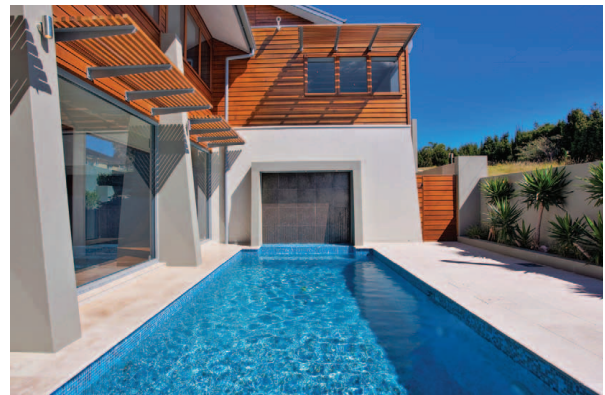
Example 14 **10** Alison and Flynn's combined age is 40. Given that Flynn is 4 years older than Alison, write an equation and use it to find Alison's age.

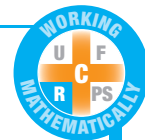
- 11** Recall that in a quadrilateral the sum of all angles is 360° . Find the values of x and y in the diagram below.



- 12** The sum of three consecutive numbers is 357.
- Use an equation to find the smallest of the three numbers.
 - What is the average of these three numbers?
 - If the sum of three consecutive numbers is 38 064, what is their average?

- 13** The breadth of a rectangular pool is 5 metres longer than the length. The perimeter of the pool is 58 metres.
- Draw a diagram of this situation.
 - Use an equation to find the pool's length.
 - Hence state the area of the pool.





- 14** The average of two numbers can be found by adding them and dividing by 2.
- If the average of x and 10 is 30, what is the value of x ?
 - If the average of x and 10 is 2, what is the value of x ?
 - If the average of x and 10 is some number R , create a formula for the value of x .
- 15** Sometimes you are given an equation to solve a puzzle, but the solution of the equation is not actually possible for the situation. Consider these five equations.
- A** $10x = 50$ **B** $8 + x = 10$ **C** $10 + x = 8$ **D** $10x = 8$ **E** $3x + 5 = x + 5$
- You are told that the number of people in a room can be determined by solving an equation. Which of these equations could be used to give a reasonable answer?
 - If the length of an insect is given by the variable x cm, which of the equations could be solved to give a reasonable value of x ?
 - Explain why equation **D** could not be used to find the number of people in a room but could be used to find the length of an insect.
 - Give an example of a puzzle that would make equation **C** reasonable.

Enrichment: Unknown numbers

- 16** Find the unknown number using equations. The answers might not be whole numbers.
- The average of a number and double the number is 25.5.
 - Adding 3 to twice a number is the same as subtracting 9 from half the number.
 - The average of a number and double the number gives the same result as adding one to the original number and then multiplying by one-third.
 - The product of 5 and a number is the same as the sum of four and twice the original number.
 - The average of 5 numbers is 7. When one more number is added, the average becomes 10. What number was added?

$$\begin{array}{r}
 5x - 40 + 2x + 10 = 25.5 \\
 7x - 30 = 180 \\
 \quad +30 \quad +20 \\
 \hline
 7x = 210 \\
 \quad \div 7 \\
 x = 30
 \end{array}$$

